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A semiquantitative theory of convective heat transfer in a heat-generating fluid

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Abstract—A semiquantitative theory of heat transfer in a heat-generating fluid within a closed volume is developed. The analysis is based on relationships derived from the condition of energy balance and on modern physical concepts of heat transfer processes in an energy-neutral fluid. Four main regimes and one asymptotic regime of heat transfer are singled out, which differ in the exponents of power in the expression relating the Nusselt number, Nu, and the modified Rayleigh number, Ra_I. In the asymptotic limit, with $Ra_I^{-1/32} \gg 1$, heat transfer to the upper horizontal part of the boundary and the curved part of the boundary facing downward obeys dependences $Nu_{up} \sim Ra_I^{7/32}$ and $Nu_{dn} \sim Ra_I^{1/4}$, respectively. In the range of values of Ra_I which is of interest with regard to the safety problem of the nuclear power engineering the established theoretical correlations are in good agreement with the experimental data. © 1998 Elsevier Science Ltd.

1. INTRODUCTION

When scenarios of core melt accidents at NPPs are analyzed, and their consequences predicted, there arises a problem of core melt retention in the reactor vessel. In this connection, it is highly important to study the processes of convective heat transfer in a heat-generating fluid within a closed volume. These processes are investigated by means of experimental and numerical [1] [2] modeling. Although the studies based on numerical methods have been quite successful, any further progress along this path towards high rates of heat release which correspond to real situations runs into considerable difficulties. Therefore, it would be useful to obtain an insight into the physical nature and qualitative correlations of possible heat transfer regimes in a heat-generating fluid.

The aim of this study is to construct a qualitative picture of heat transfer in a heat-generating fluid within the entire power range of interest, proceeding from the modern physical concepts of convective heat transfer. In Section 2, a set of relationships is derived from the condition of energy balance. Section 3 is devoted to semiquantitative correlations of heat transfer corresponding to natural convective flow regimes which change depending on the rate of heat release. Also in this section the results of the study are compared with the known experimental data.

2. ENERGY BALANCE CONDITION

Let us consider heat transfer in a heat-generating fluid occupying a volume V which has a height H and a rigid isothermal boundary S shown schematically in Fig. 1.

A horizontal plane passing through a point which corresponds to the maximum value of the time-averaged temperature of the fluid divides the entire volume V into two parts of approximately the same height. Due to the inverse distribution of temperature a situation close to the conditions of Rayleigh–Benard (RB) convection arises in the upper part, V_+, having a height H_+. The corresponding flow of the fluid provides heat transfer to the upper horizontal part of the boundary having an area S_{up}. Heat transfer to the curved part of the boundary facing downward, and having an area S_{dn}, is due to the presence of a boundary layer (BL), which is thin in comparison with H and in which the fluid flows down. At such time, a stable stratified distribution of temperature appears inside the lower part V_- of the volume V ($V_- = V - V_+$) in the presence of a return upward flow of the liquid.

The stationary condition of energy balance has the form:

$$\frac{\lambda \Delta T}{H}\int_s \mathrm{d}S Nu = QV \quad S = S_{up} + S_{dn} \qquad (1)$$

where $\Delta T = T_{max} - T_s$ is an excess of the maximum temperature, T_{max}, within the volume V over the boundary temperature, T_S; Q is power release density. Expressing these values through the Rayleigh number and the modified Rayleigh number we obtain the relationship

$$Ra\overline{Nu} = Ra_I \qquad (2)$$

in which

$$\overline{Nu} = \frac{H}{V}(S_{up}Nu_{up} + S_{dn}Nu_{dn}) \qquad (3)$$

NOMENCLATURE

A	aspect ratio of the V_+ domain, $A = D/H_+$
BL	boundary layer
D	horizontal dimension of the V_+ domain
g	acceleration due to gravity
H	height of the volume V
H_+	height of the V_+ domain
H_-	height of the V_- domain
k	thermal diffusivity
Nu	Nusselt number, $Nu = (H/\lambda\Delta T)q$
Pr	Prandtl number, $Pr = \nu/k$
Q	power release density
q	heat flux density to the boundary
Ra	Rayleigh number, $Ra = g\alpha\Delta TH^3/\nu k$
Ra_I	modified Rayleigh number, $Ra_I = g\alpha QH^5/\nu k\lambda$
Ra_{C1}, Ra_{C2}, Ra_{C3}	critical values of Ra_+ in RB convection
Ra^*	critical value of Ra in BL
RB	Rayleigh–Benard
S	area of the volume V boundary
T	fluid temperature
T_{max}	temperature maximum value in the volume V
T_S	temperature of the boundary
ΔT	$= T_{max} - T_S$
V	volume occupied by fluid
V_+, V_-	upper and bottom parts of volume V.

Greek symbols

α	thermal expansion coefficient
β, β_i, γ_i, ε	power exponents in (5)
λ	thermal conductivity
ν	kinematic viscosity.

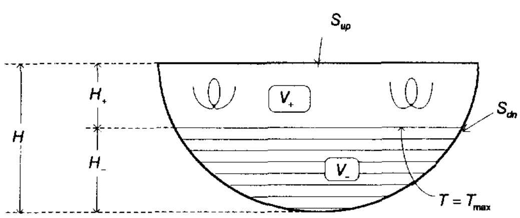

Fig. 1. Geometry of the problem.

$$Nu_i = \frac{1}{S_i}\int_{S_i} dS Nu, \quad i = \text{up}, \text{dn}. \qquad (4)$$

Theoretical dependences $Nu_i(Ra)$ and the corresponding results of experimental and numerical modeling are usually expressed by power functions. Let us define power exponents β_{up}, β_{dn}, β, γ_{up}, γ_{dn}, and ε through relationships

$$Nu_i \sim Ra^{\beta_i}, \quad \overline{Nu} \sim Ra^{\beta}, \quad Ra \sim Ra_I^{\varepsilon}, \quad Nu_i \sim Ra_I^{\gamma_i}. \qquad (5)$$

Substitution of eqn (5) into eqn (2) leads to the establishment of the following important constraints between the power exponents:

$$\varepsilon = (1+\beta)^{-1} \qquad (6)$$

$$\gamma_i = \frac{\beta_i}{1+\beta}. \qquad (7)$$

Now we have to find power exponents γ_{up}, γ_{dn} and ε disregarding numerical coefficients of the order of unity in eqn (5) as the determination of these coefficients is outside the scope of the semiquantitative theory elaborated here. We assume that $Pr \gtrsim 1$. Note that dependences (5) with constant exponent may hold true within limited variation ranges of the master parameter Ra_I depending on definite combinations of heat transfer regimes at the boundary sections S_{up} and S_{dn}. When the regimes change, the power exponents also change (abruptly or gradually).

3. SEMIQUANTITATIVE CORRELATIONS FOR HEAT TRANSFER

In the analysis of heat transfer in a heat-generating fluid we will proceed from two analogies with heat transfer in energy-neutral fluids (i.e. fluids which do not have internal heat sources). The first analogy is based on the similarity between the processes occurring in the region V_+ of the problem discussed here and RB convection. The second analogy is based on the similarity between boundary layers (BL) in heat-generating and energy-neutral fluids. The reasons which warrant such analogies are as follows.

With $Ra \gg 1$, thermal resistance is mainly accounted for by narrow wall-adjoining layers of the fluid having a thickness much smaller than H. Therefore, referred to the unit area of the wall-adjoining thermal layer the rate of heat release in this layer is small compared to the density of the thermal flux passing through it and, consequently, heat release in these layers produces practically no effect on their structure and, accordingly, on the heat transfer characteristics. Additional proof of close similarity between convection in the region V_+ of a heat-generating fluid and RB convection based on the available experimental data will be given elsewhere in this section. Let us recall the known characteristics of convective heat transfer in an energy-neutral fluid which are used as prototypes for a heat-generating fluid.

RB convection corresponding to the horizontal layer of the fluid heated from below has the following heat transfer regimes differing in the values of exponent β_{RB} in the relationship $Nu_{RB} \sim Ra_+^{\beta_{RB}}$, where $Ra_+ = Ra(H \to H_+)$. With $Ra_{C1} < Ra_+ < Ra_{C2}$, there is a laminar flow with $\beta_{RB} = 1/4$ [3]. With $Ra_{C2} < Ra_+ < Ra_{C3}$, a so-called soft turbulence regime is realized, for which $\beta_{RB} = 1/3$ [4]. With $Ra_+ > Ra_{C3}$, the soft regime is replaced by a hard turbulence regime, for which $\beta_{RB} = 2/7$ [4]. Critical values of Ra_{C1}, Ra_{C2}, Ra_{C3} depend on the aspect ratio $A = D/H_+$, where D is the horizontal dimension of the RB cell. With $A > 1$, $Ra_{C1} \sim 10^3$. If $A \approx 1$, $Ra_{C2} = 2 \cdot 10^5$ and $Ra_{C3} = 4 \cdot 10^7$ [4]. If $A = 6.5$, $Ra_{C3} \cong 10^4$ [5]. These data suggest that with the values of A appreciably larger than unity, when Ra increases, a laminar flow in the RB cell directly changes to a hard turbulence regime, without passing through a soft regime.

The vertical boundary layer in an energy-neutral fluid has two regimes. These are a laminar regime with $\beta_{BL} = 1/4$ [6] and a turbulence regime with $\beta_{BL} = 1/3$ [7], where β_{BL} is the exponent in the relationship $Nu_{BL} \sim Ra^{\beta_{BL}}$. The transition from one regime to the other corresponds to the critical value of $Ra = Ra^*$, which depends on the Pr number. Theoretically, with $Pr > 1$, according to ref. [8] this dependence has the form $Ra^* \sim Pr^2$. However, the numerical coefficient in this relationship can vary by one and a half orders of magnitude from experiment to experiment, and this may render the dependence of Ra^* on Pr insignificant when the Pr number varies within a limited range.

We further assume that each heat transfer regime in a heat-generating fluid is a combination of the regimes in the region V_+ and in the boundary layer which correspond to the regimes in the prototype systems. Therefore, taking into account the definition of the Nu number we have:

$$Nu_{up} \sim \frac{H}{H_+} Ra_+^{\beta_{RB}} = \left(\frac{H}{H_+}\right)^{1-3\beta_{RB}} Ra^{\beta_{RB}},$$

$$Nu_{dn} \sim Ra^{\beta_{BL}}. \tag{8}$$

Because the interface between the regions V_+ and V_- corresponds to the maximum temperature within the volume V, the difference between the heights H_+ and H_- is closely connected with the difference between Nu_{up} and Nu_{dn}. In virtue of inequalities

$$|\beta_{RB} - \beta_{BL}| \ll \beta_{BL}, \quad 1 - 3\beta_{RB} \ll 1 \tag{9}$$

[the second one conditions weak dependence on H/H_+ in the first relationship (8)] and owing to the fact that within the range of the greatest practical importance, $Ra_I < 10^{16}$, the intervals, within which heat transfer regimes are realized with respect to the Ra_I number, are comparatively small, it can be considered that here $H_+ \approx H/2$. Therefore, when recalculating boundaries of the intervals corresponding to various convective regimes from the prototype systems to the problem under consideration we can take $Ra \approx 10Ra_+$. In the same range of values of the Ra_I number, due to eqns (3) and (5) the power exponent β satisfies the relationship

$$\beta = \frac{\beta_{up} + \beta_{dn}}{2} \pm \frac{|\beta_{up} - \beta_{dn}|}{2}. \tag{10}$$

Taking into account the relationships (8) and (10) and proceeding from what has been said above concerning the characteristics of heat transfer in an energy-neutral fluid four main heat transfer regimes can be singled out in a heat-generating fluid with the following interval boundaries and characteristics [flow regimes in the region V_+ and in BL, exponents β, ε, γ_i defined by relationships (5)]:

I $Ra_I^{(1)} < Ra_I < Ra_I^{(2)}$: laminar convection in V_+ domain and in BL

$$\beta = 0.25, \quad \varepsilon = 0.8, \quad \gamma_{up} = \gamma_{dn} = 0.2. \tag{11}$$

II $Ra_I^{(2)} < Ra_I < Ra_I^{(3)}$: soft turbulence in V_+, laminar BL

$$\beta = 0.29 \pm 0.04, \qquad \varepsilon = 0.775 \pm 0.025,$$
$$\gamma_{up} = 0.263 \pm 0.008, \quad \gamma_{dn} = 0.195 \pm 0.005. \tag{12}$$

III $Ra_I^{(3)} < Ra_I < Ra_I^{(4)}$: hard turbulence in V_+, laminar BL

$$\beta = 0.27 \pm 0.02, \qquad \varepsilon = 0.79 \pm 0.03,$$
$$\gamma_{up} = 0.225 \pm 0.004, \quad \gamma_{dn} = 0.197 \pm 0.003. \tag{13}$$

IV $Ra_I^{(4)} < Ra_I < 10^2 \cdot Ra_I^{(4)}$: hard turbulence in V_+, combination of laminar and turbulent flow regimes in BL

$$\beta = 0.31 \pm 0.025, \qquad \varepsilon = 0.765 \pm 0.015,$$
$$\gamma_{up} = 0.218 \pm 0.004, \quad \gamma_{dn} = 0.255 \pm 0.005. \tag{14}$$

(the values of these exponents correspond to the end of interval IV).

In accordance with the aforementioned characteristics of heat transfer in an energy-neutral fluid and taking into account the relationships (10), (5) and (6) the boundaries of regimes I–IV in (11)–(14) are defined as follows:

$$Ra_I^{(1)} \cong 10^5, \quad Ra_I^{(2)} \cong 15(Ra_{c2})^{1.25}$$
$$Ra_I^{(3)} \cong 15(Ra_{c3})^{1.29}, \quad Ra_I^{(4)} \cong 15(Ra_*)^{1.27}. \tag{15}$$

The values of $Ra_I^{(2)}$ and $Ra_I^{(3)}$ depend on the aspect ratio A for the region V_+ and the value of $Ra_I^{(4)}$ depends on the Prandtl number. At sufficiently high values of A the regime II may be absent altogether and, in this case, regime III directly follows regime I.

Some uncertainty in the dependence of Ra^* on the Prandtl number mentioned above affects the boundary value $Ra_I^{(4)}$. For example, for water pools this value is equal to $Ra_I^{(4)} \cong 2.5 \cdot 10^{13}$ [9].

Off all regimes, I–IV, regime IV has the greatest importance so far as the safety problem of the nuclear power engineering is concerned. In this regime the boundary layer changes from a laminar to a turbulent state. When $Ra_I > 10^2 \cdot Ra_I^{(4)}$, the turbulent section of the boundary layer becomes predominant. Taking into account numerical coefficients in the dependences $Nu(Ra)$ for a laminar and a turbulent boundary layer of an energy-neutral fluid, the expression $Nu_{dn}(Ra)$ in regime IV of a heat-generating fluid has the form:

$$Nu_{dn} \approx 0.68 Ra_*^{1/4} + 0.15[Ra^{1/3} - Ra_*^{1/3}]. \quad (16)$$

Within the flow readjustment interval of the boundary layer the rate of rise of the Nu_{dn} number depending on Ra_I increases. If, as before, we describe this dependence through the relationship of (5) the exponent γ_{dn} will be a function Ra_I number corresponding to a logarithmic derivative of the expression (16). For the value of the exponent $\bar{\gamma}_{dn}$ averaged over the flow readjustment interval we have

$$\bar{\gamma}_{dn} = \frac{\Delta \ln Nu_{dn}}{\Delta \ln Ra_I} \quad (17)$$

where $\Delta \ln Nu_{dn}$ is change in the value of $\ln Nu_{dn}$ over the averaging interval from $Ra_I^{(4)}$ to $Ra_I^{(4)} \exp[\Delta \ln Ra_I]$. Taking $Ra_I^{(4)} = 2.5 \cdot 10^{13}$, which is true for experiments with water, and $\Delta \ln Ra_I = \ln 10^2$ we obtain:

$$\bar{\gamma}_{dn} \approx 0.36. \quad (18)$$

As the temperature maximum in the volume V corresponds to the interface between the regions V_+ and V_-, with $\beta_{up} \neq \beta_{dn}$ and Ra_I growing, the interface shifts in the direction in which T_{max} relatively decreases. Up to the values of Ra_I equal to $Ra_I^{(4)}$ it takes place $\beta_{up} \geqslant \beta_{dn}$, and with the growth of Ra_I in this interval the interface slowly moves downward. After transition of the boundary-layer flow to a turbulent state β_{dn} becomes higher than β_{up} and, as Ra_I grows still more, no changes in the flow regime are anticipated. This means that the interface is moving upward leading eventually to an asymptotic heat transfer regime: $V_+ \ll V$, $H_+ \ll H$. In this case, practically all heat released in the region V_+ is transferred through the upper section of the boundary, S_{up}. Then, on the basis of the relationships (5)–(8) and the condition of energy balance written separately for the region V_+ we obtain:

$$\beta = \tfrac{1}{3}, \quad \varepsilon = \tfrac{3}{4}, \quad \gamma_{dn} = \tfrac{1}{4}, \quad \gamma_{up} = \tfrac{7}{32} \approx 0.219. \quad (19)$$

These values of power exponents determine the correlations of the asymptotic heat transfer regime. The condition for realization of this regime has the form:

$$Ra_I^{-1/32} \ll 1. \quad (20)$$

Now, let us compare the semiquantitative correlations established here with experimental results. The best known correlations of experimental data obtained from modeling heat transfer in the melt are as follows:

Kulacki–Emara [10]:

$$Nu_{up} \sim Ra_I^{0.227} \quad 2 \cdot 10^4 < Ra_I < 4.4 \cdot 10^{12} \quad (21)$$

Steinberner–Reineke [11]:

$$Nu_{up} \sim Ra_I^{0.233}$$

$$Nu_{dn} \sim Ra_I^{0.19} \quad 10^7 < Ra_I < 3 \cdot 10^{13} \quad (22)$$

Jahn–Reineke [12]:

$$Nu_{up} \sim Ra_I^{0.23}$$

$$Nu_{dn} \sim Ra_I^{0.18} \quad 10^7 < Ra_I < 5 \cdot 10^{10} \quad (23)$$

Theofanous–Liu [13]:

$$Nu_{dn} \sim Ra_I^{0.27} \quad 10^{12} < Ra_I < 3 \cdot 10^{13}$$

$$Nu_{dn} \sim Ra_I^{0.35} \quad 3 \cdot 10^{13} < Ra_I < 7 \cdot 10^{14}. \quad (24)$$

Comparison of power exponents in these dependences with exponents γ_{up}, γ_{dn} in (11)–(14) suggests the following conclusions. The experimental data concerning γ_{up} indicate that, most likely, theoretical heat transfer regime II is not realized under the conditions of the experiments [10–13]. This conclusion is based on the following. As was mentioned above, with moderately high values of Ra_I $H_+ \sim H/2$. On the other hand, in these experiments the ratio of the characteristic dimension of the volume V in the horizontal direction to the height H is of the order of 2. Therefore, the aspect ratio for the volume V_+ is $A \sim 4$. In accordance with [5] this may lead to the disappearance of a soft turbulence regime in the volume V_+. In this case one can say that there is good agreement between theoretical predictions (11), (13), (14) and experimental data (21)–(24) regarding the exponent γ_{up}.

The theoretical and experimental values of the exponent γ_{dn} are in good agreement when $10^5 < Ra_I < 10^{11}$. A substantial growth of this exponent when $Ra_I > 3 \cdot 10^{13}$, which was observed in the ACOPO experiment [13], may be due to the readjustment of the boundary-layer flow from a laminar to a turbulent state. Our value of $\bar{\gamma}_{dn} \sim 0.36$ (18) is in good agreement with the experimental correlation (24).

In conclusion, we will again dwell on the question of whether it is possible experimentally to corroborate the analogy of heat transfer mechanisms in the region V_+ of a heat-generating fluid and in RB convection. Heat transfer in a heat-generating fluid occupying the volume of a horizontal plane-parallel layer with a heat-insulated lower boundary and an isothermal upper boundary was modeled in the Kulacki–Emara experiment [10]. In this experiment the aspect ratio corresponded to $4 \leqslant A \leqslant 20$. Considering that the experiment [10] coincided with RB convection in respect of the geometry and assuming that there is an analogy between the heat transfer processes in both cases, from eqn (7) we obtain the following relationship between the exponent $\gamma = \gamma_{up}$ for the experiment [10] and $\beta = \beta_{RB}$ for RB convection:

$$\gamma_{up} = \frac{\beta_{RB}}{1+\beta_{RB}}. \tag{25}$$

The range of values of the Rayleigh number [see eqn (21)] and the aspect ratio in the experiment [10] correspond to the conditions of a hard turbulence regime for which $\beta_{RB} = 2/7$ [4]. By substituting this value in eqn (25) we obtain the theoretical value of the exponent γ_{up} equal to $\gamma_{up}^{theor} \cong 0.222$. The fact that this value is close to the value $\gamma_{up}^{exp} = 0.227$ from ref. [10] can be regarded as experimental proof of the analogy between heat transfer mechanisms in the region V_+ of a heat-generating fluid and in RB convection.

4. CONCLUSIONS

The main conclusions from this study are as follows. We have determined semiquantitative correlations of heat transfer in a heat-generating fluid and singled out four main regimes and one asymptotic regime of heat transfer. In regime IV, the most important one for the safety problem in the nuclear power engineering, the boundary-layer flow changes from a laminar to a turbulent state. This substantially increases the rate of growth of heat transfer depending on the Rayleigh number in the curved part of the boundary which faces downward. In the asymptotic regime, when $Ra_I^{-1/32} \ll 1$, downward heat transfer prevails. The theoretical dependences obtained in the study are in good agreement with the experiment.

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